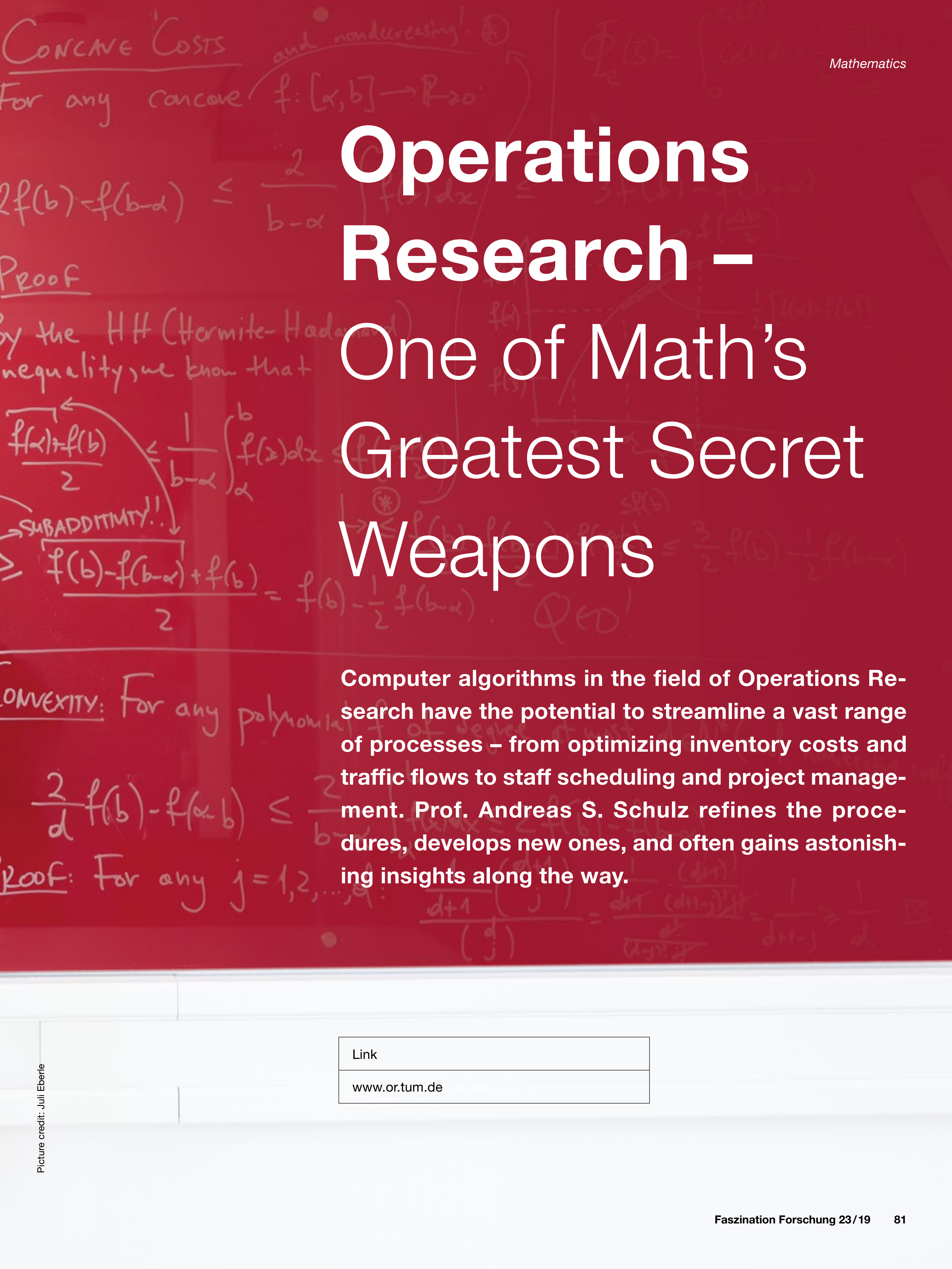

Operations Research – One of Math's Greatest Secret Weapons

Computer algorithms in the field of Operations Research have the potential to streamline a vast range of processes – from optimizing inventory costs and traffic flows to staff scheduling and project management. Prof. Andreas S. Schulz refines the procedures, develops new ones, and often gains astonishing insights along the way.

Link
www.or.tum.de

Picture credit: Juli Eberle

Kurzfassung

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Operations Research – eine mathematische Wunderwaffe

Es handelt sich um einen Zweig der Mathematik, der sich vor allem mit der Modellierung und Optimierung wirtschaftlicher Abläufe befasst. Aus einer Vielzahl von Möglichkeiten können Algorithmen die jeweils beste auswählen und zur Entscheidungsfindung in Unternehmen vorschlagen. Andreas S. Schulz, Alexander von Humboldt-Professor für Operations Research an der TUM, entwickelt die dafür nötigen Verfahren weiter, erfindet neue und entdeckt bei der dahinter liegenden Theorie manchmal verblüffende Ergebnisse. So konnte er zum Beispiel nachweisen, dass ein Problem aus der Lagerhaltung mindestens so schwer zu lösen ist wie die Faktorisierung ganzer Zahlen. Oder dass ein vernetztes System, in dem jeder nur seinem eigenen Nutzen folgt, nur wenig schlechter funktioniert als eines, in dem alle dem allgemeinen Wohl folgen. Schulz engagiert sich nicht nur für ökonomische Fragen, sondern wendet seine Verfahren auch auf soziale und humanitäre Projekte an. □

The many influences of mathematics on our everyday lives are exemplified by the work carried out at the Chair of Operations Research in a building on Karlstrasse in Munich. While the shop on the building's ground floor sells shiny helium balloons floating in the air, up on the 6th floor, researchers are thinking about ways of getting their mathematical ideas off the ground. Specifically, the researchers focus on developing and improving algorithms that optimize complex processes inherent to a wide range of businesses and industries. The number of possible executions for a complex process can be extremely large, so the objective is often to efficiently find the best one in terms of time and resources. "We seek methods that can deal with both huge volumes of data and with uncertainty," explains the Alexander von Humboldt Professor Andreas S. Schulz, who has been head of this chair and an interdisciplinary research center between the Department of Mathematics and the School of Management since 2015. "We also take challenging circumstances into consideration, such as dynamically changing environments, with privately held information and multiple – sometimes conflicting – objectives."

The optimal route

The traveling salesperson problem (TSP) is regarded as a classic problem in Operations Research (OR). The objective is to determine the order in which a salesperson should visit their customers so as to minimize the distance traveled. "The beauty of this problem is that it relates to the real world, can be easily grasped, and everyone is quick in proposing ideas for solving it," says Schulz. "The fascinating aspect for me, though, is that this problem is mathematically so difficult that decades of research have so far failed to come up with a general solution method that finds optimal solutions to all instances efficiently. It is also a prototype for similar tasks where you have to choose a best alternative from a multitude of possibilities."

When the number of customers is small, all routes can be explicitly listed and easily compared. However, the number of potential routes increases rapidly as the customer base expands: with five customers, there are 24 possible options. With six, there are 120. If the traveling salesperson has 13 customers, the number of possible routes already exceeds the odds of getting six numbers out of ▷

customers	number of possible routes
5	24
6	120
25	620 448 401 733 239 439 360 000

The traveling salesperson problem (TSP) has the objective of determining the order in which a salesperson should visit their customers so as to minimize the distance traveled. The picture shows two possible routes (blue and red). The TSP is a challenging mathematical problem, not only because the number of potential routes increases rapidly as the customer base expands (see left).

Graphics: ediundsepp

49 in the lottery. With 25 customers, there are sextillions of potential routes. The most powerful supercomputers would take months to calculate all of these possible combinations. Clearly, with several hundred customers the problem cannot be solved simply by listing and comparing the efficiency of individual routes, and a more strategic approach is needed.

The traveling salesperson problem occurs in many different contexts, such as the logistics of parcel deliveries, organization of flight plans, and emergency room scheduling, as well as the planning of inventory picking from high-bay warehouses, and even sequencing drill holes in circuit boards. Nowadays, there are mathematical algorithms capable of finding good or optimal solutions for many of these practical problems – for example, the branch-and-cut method. However, there is no known method that can always find an optimum efficiently.

“The question of why we have not yet managed to discover an efficient exact algorithm to the traveling salesman problem is central to OR and theoretical computer science,” explains Schulz. “It is no surprise that in the year 2000, the Clay Mathematics Institute in Massachusetts announced one million dollars in prize money to anyone who could develop what is called a polynomial-time algorithm for solving the TSP, or else prove that no such algorithm exists.”

Military origins

The origins of OR can be traced back to methods used by the British and Americans in World War II and subsequently in the Korean War to improve military strategies and weapons deployment. It was not until the 1950s that non-military companies started to use these optimization methods for their own purposes. Soon after that, an academic community began to develop around the subject. Since then, several Nobel Prizes in Economic Sciences have been awarded to researchers for their discoveries in the field of OR (see box).

Nobel Prizes in Economic Sciences for OR problems

1969 Jan Tinbergen jointly with Ragnar Frisch “for having developed and applied dynamic models for the analysis of economic processes”.

1975 Leonid V. Kantorovich and Tjalling C. Koopmans “for their contributions to the theory of optimum allocation of resources”.

1990 Harry M. Markowitz jointly with William F. Sharpe and Merton H. Miller “for the application of mathematical or computer techniques to practical problems, particularly problems of business decisions under uncertainty”.

1994 John C. Harsanyi jointly with John F. Nash Jr. and Reinhard Selten “for their pioneering analysis of equilibria in the theory of non-cooperative games”.

2012 Alvin E. Roth and Lloyd S. Shapley “for the theory of stable allocations and the practice of market design”.

The search for an optimal warehousing strategy is deceptively difficult. If you have two or more different products that may be ordered together at a discount but do not have to be, nobody knows of an efficient algorithm to identify an optimal solution.

"Sometimes I hear about a new method and consider whether it would work for one of my own mathematical problems," says Professor Schulz.

Picture credit: supersizer/iStock, Juli Eberle

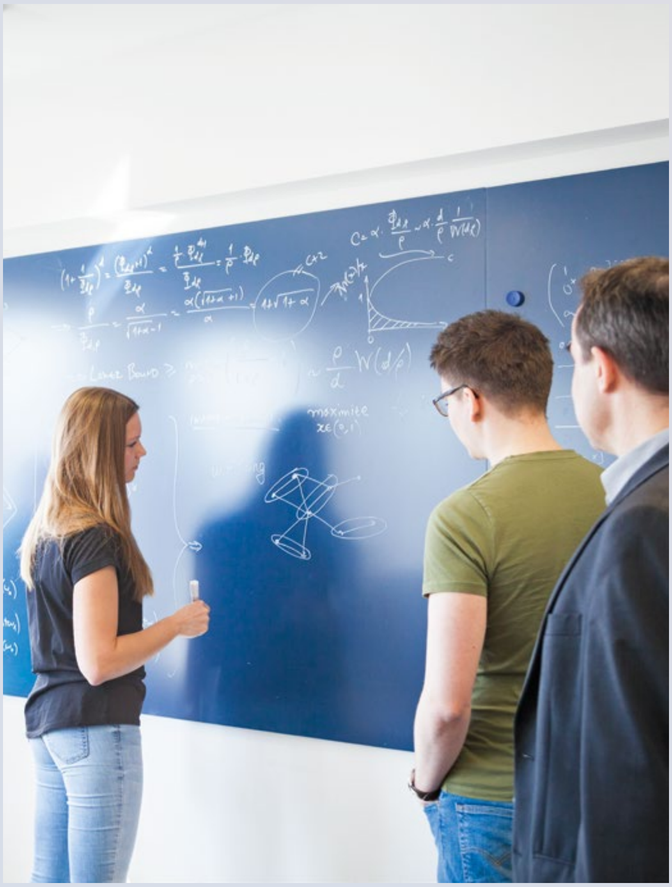
Nowadays, you can buy software programs for warehousing, business processes and logistics. Schools and railway companies use this kind of software to compile their timetables. The success of industry giants like Amazon and Google is closely tied to the use of mathematical methods. In contrast, however, small and medium-sized enterprises still tend to be unaware of the huge potential for improvement that OR can bring. Another factor for the untapped use of OR in smaller enterprises could be that OR requires a certain level of background knowledge. "Standard software is available," the professor points out, "but I have found that every situation has its own particular set of circumstances and one size does not fit all. You often have to start by developing a suitable model that captures the specifics of the case in question."

Schulz is one of the world's leading academics in the field of Operations Research. He completed his Master's degree and PhD at the Technical University of Berlin, and captured the attention of the academic community early on with his doctoral thesis. At the young age of 29, he was appointed Professor of Mathematics of Operations Research at the Massachusetts Institute of Technology (MIT) in Cambridge, USA, where he later took up the Patrick J. McGovern Chair of Management Science. ▷

He remained at MIT for 17 years with occasional visiting positions at other universities before accepting an offer from TUM in 2015. He has received prizes and awards for his scientific work almost on a yearly basis, and his discoveries have not only solved problems that mathematicians have grappled with for a long time, but they have also had significant practical implications.

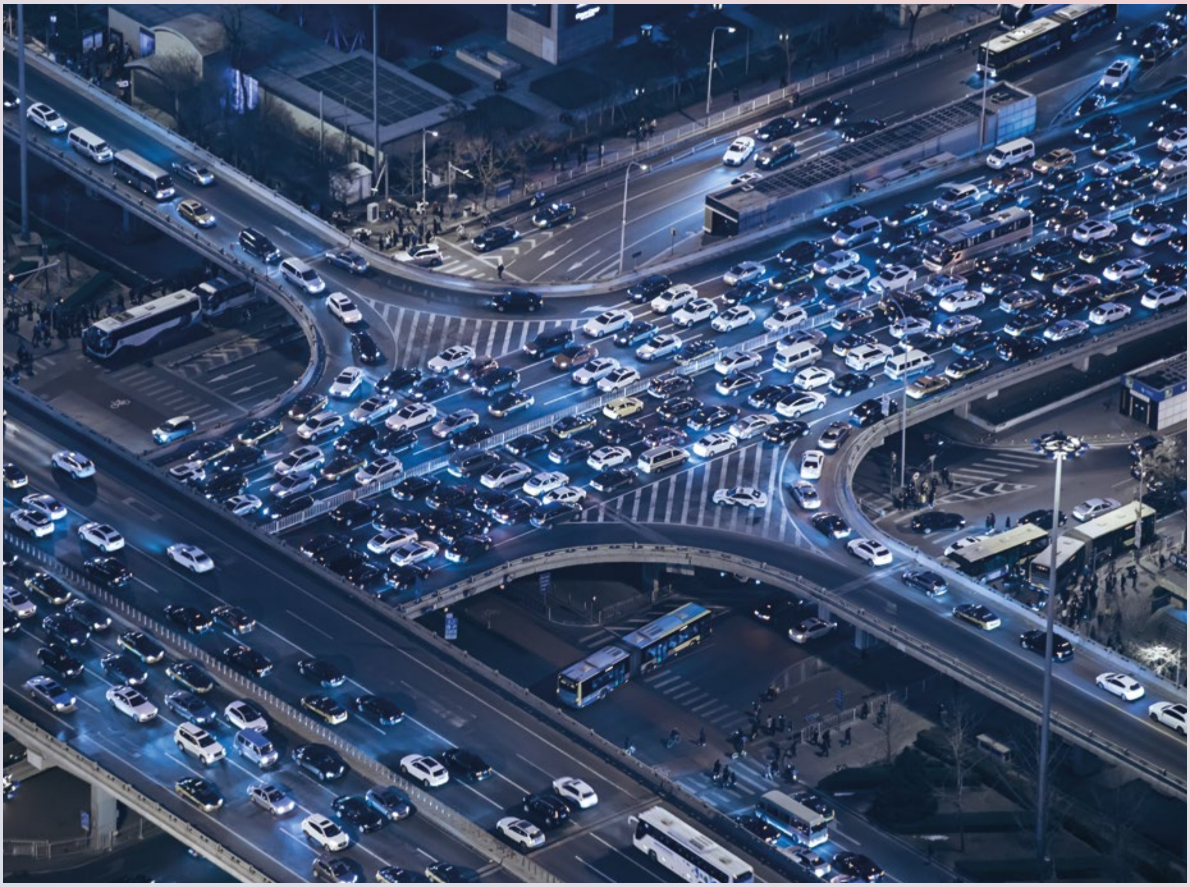
Ideas in the shower

Many of his best ideas come to him through discussions with colleagues: “A large number of proposals and problems are aired at symposiums for example. Sometimes you encounter a problem and think: Aha, I can think of a technique that might work for that. Or else I hear about a new method and consider whether it would work for one of my own mathematical problems. I was once in a meeting where all three speakers were talking about their individual topic, yet at the end I felt that they had all said the same thing without even noticing. The following year, I developed a common theory for all of these contributions.”
The flow of ideas then continues at home: “Guests sometimes come to the institute to give a talk. I latch on to something that fascinates me and it sticks in my memory,” says Schulz. “Another idea then comes to me, perhaps while walking or having a shower. In mathematics, the subconscious plays a very important role.” At this stage, he uses the computer only to test a conjecture or idea.
This is also the methodology Schulz recommends to his students and doctoral candidates: “I often say to my group: You need to have a deep understanding of the subject because the solutions will then eventually present themselves. You cannot consciously work towards it. We can set out to tackle major unresolved problems, but the solution is unlikely to come within a year or two. In the meantime, we will come across other interesting questions, and will be able to present partial solutions, at the very least.”



Many of his best ideas come to Andreas S. Schulz through discussions with colleagues.

This has happened, for example, in the search for an optimal warehousing strategy – another problem that seems straightforward at first glance, but is deceptively difficult. If you look at the ordering costs and compare them with the warehousing costs, it is easy to calculate the delivery intervals required to keep the overall costs as low as possible for a single product. But if you have two different products that may be ordered together at a discount but do not have to be, nobody knows of an efficient algorithm to identify an optimal solution. This is precisely the type of challenge that absorbs a mathematician like Schulz.
He and one of his PhD students succeeded in showing that the problem is at least as difficult to solve as the factorization of integers. Integer factorization involves decomposing an integer into a product of smaller integers, for example 39 = 3 x 13. Calculating the product of two large prime numbers is unproblematic. However, if the product has hundreds of digits, then the reverse procedure of finding two factors is extremely difficult.



Methods of OR are also suitable for the planning and optimization of new traffic routes.

The computing effort required quickly exceeds the realistic realm of possibility. Because of this, the factorization of very large numbers is a security component in the RSA encryption processes frequently used today.

The price of anarchy

Another problem, which is highly relevant in everyday life and whose solution sometimes defies common sense, was initially studied over 20 years ago. The question goes: If you have a networked system where everyone does what is best for them personally, how much more unfavorably does the overall system function in comparison to a situation where everyone does what is best for the common interest? Or in other words: What is the price of anarchy?

For example, in traffic flows it is naturally assumed that all drivers are taking the route which they believe is the quickest, and this often leads to traffic jams on many major roadways. However, it could be possible to avoid congestion if traffic was directed so that not all cars take their assumed quickest route, thus improving the total travel time. ▷

Braess Paradox

The Braess Paradox is an unexpected result from network theory. It states that adding capacity could actually slow down the speed of the network. Applied to highways, the Braess Paradox means that the addition of one road can make the average journey time longer, or that closing some roads could speed up traffic. This effect was observed in New York City, for example, on the occasion when 42nd Street near Times Square was closed to traffic for Earth Day on April 22, 1990. Everybody had predicted that traffic chaos would ensue, but they were mistaken – the traffic distributed itself evenly onto the other streets.

Picture credit: Juli Eberle, Wenjie Dong/istock

> *“You need to have a deep understanding of the subject because the solutions will then eventually present themselves.”*
>
> *Andreas S. Schulz*

By modeling these systems mathematically, the surprising insight was found that the addition of one road can make the journey time longer. This effect is known as the Braess Paradox, and was discovered in 1968 by the German mathematician Dietrich Braess (see box). That this effect can be observed in real life was proven in New York City, for example, on the occasion when 42nd Street was closed to traffic for Earth Day on April 22, 1990. Everybody had predicted that traffic chaos would ensue, but they were mistaken – the traffic distributed itself evenly onto the other streets.

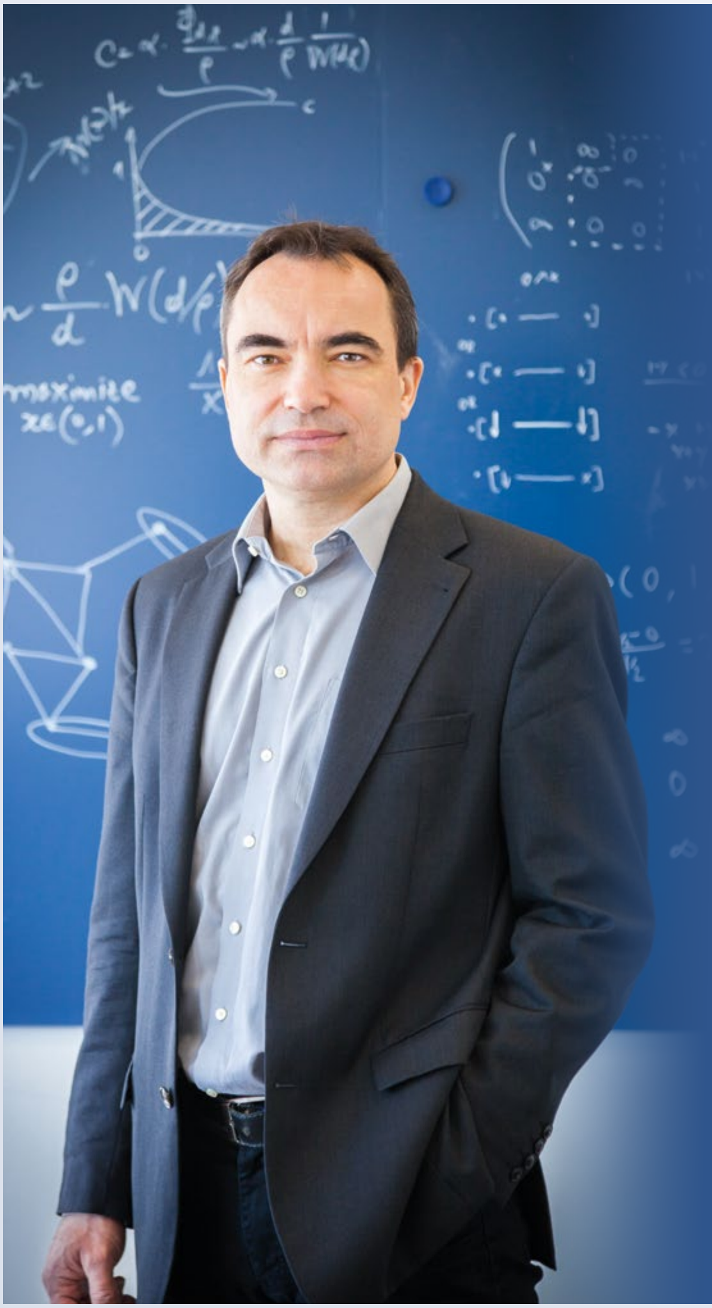
Schulz attempted to analyze this question, and for the price of anarchy he found a “very nice geometric proof which is now included in standard textbooks.” He was able to demonstrate that with a particular type of driving behavior, the worst possible solution is only 33 percent worse than the best possible one. “Those are idealized assumptions, however,” he admits. “You can make a few refinements for real-life road networks. Then you determine that you lose 36 percent in the worst-case scenario.”

It is possible to significantly reduce costs with OR. An example comes from Jannik Matuschke, a former member of Schulz’s group, who was appointed as an Assistant Professor at the Catholic University of Leuven in January. His doctoral thesis shows that a parcel delivery service in Germany was able to achieve a 14 percent reduction in costs over a year, corresponding to EUR 1.6 million, with an OR method specifically developed for this company. Schulz is keen, however, for his methodologies to be used for more than just maximizing the profits of businesses. He wants to use them to give something back to society and to support non-profit organizations. For example, they could significantly improve distribution of aid in disaster zones or revolutionize the allocation of donor kidneys.

Many people are willing to donate a kidney to a relative, but discover that they are biologically incompatible. Sometimes there are also altruistic donors who are willing to donate one of their kidneys to a complete stranger. OR makes it possible to build chains of the following type: if a patient receives an altruistic donation, that patient’s incompatible donor donates to another patient, and so on. With OR optimization methods it is possible to arrive at a situation where many more people than before receive a compatible kidney. “This has already been put into practice in the USA,” says Schulz. “In Germany, donations to strangers are virtually banned, so we are not able to realize such a scheme here just yet. There is of course always a chance that the law will be amended slightly to make this possible.”

Looking at the bigger picture, Schulz sees a new era dawning for OR: “Due to the ever-increasing overlap of computer science, mathematics and economics, new opportunities and applications naturally arise. We can continue to develop our methods by incorporating intangible factors such as incentives and uncertainties. This will put us in an even better position to tackle some of the critical challenges that society is facing today.” ■

Brigitte Röthlein



Prof. Andreas S. Schulz

A mathematician reaching out to economics

Prof. Andreas S. Schulz, born in 1969, studied applied mathematics at the Technical University of Berlin, where he completed his PhD in 1996. He spent two years there as a research assistant before accepting an appointment at the Massachusetts Institute of Technology (MIT) in Cambridge, USA. After starting as an assistant professor, he went on to become Patrick J. McGovern Professor of Management Science and Professor of Mathematics of Operations Research. Having received a Humboldt Research Award, he accepted a visiting position at TUM in 2011, and returned to his alma mater in Berlin for a research stay in 2013. He has been a visiting professor at several universities, including the University of British Columbia, the Swiss Federal Institute of Technology (ETH) Zurich, and the University of Maastricht. He is also a founding member of the Junge Akademie (Young Academy) in Berlin, Germany.

Picture credit: Juli Eberle